Landmarks via distances

• Landmarks via distances:
  – Choose *landmarks* according to *biological* criteria.
  – Choose interlandmark *distances* according to *geometric* criteria.
• Leads to propositions about the \textit{placement} and \textit{use} of interlandmark distances:
  – Distances should be placed relatively \textit{symmetrically} and \textit{regularly} among landmarks:
    • Spacing and orientation.
  – Allow for archiving \textarrow{} reconstruction \textarrow{} prediction.
  – Allow for averaging and standardization of forms.
  – Allow for error compensation, via redundancy.
  – Biologically informative “characters” should derive from analysis (e.g., by optimal contrasts) rather than being assumed beforehand.
• Three possible geometric protocols for archiving point configurations with interpoint distances:

1. All possible pairwise distances:
   • Completely (globally) redundant.

2. Triangulation:
   • Many alternate triangulation schemes for a given set of landmarks.
   • Some better than others?

3. Trusses:
   • Small subset of all possible pairwise distances.
   • Triangulation, with redundancy.
(1) All possible pairwise distances:
   - Global redundancy.
   - For $n$ landmarks, requires $\frac{1}{2}n(n-1)$ distances.
   - Basis of EDMA (Euclidean distance matrix analysis):
     • Form a square symmetric distance matrix for individual $k$:
       $$D_k = \begin{bmatrix}
         0 & d_{12} & d_{13} & \ldots & d_{1p} \\
         d_{21} & 0 & d_{23} & \ldots & d_{2p} \\
         d_{31} & d_{32} & 0 & \ldots & d_{3p} \\
         \vdots & \vdots & \vdots & \ddots & \vdots \\
         d_{p1} & d_{p2} & d_{p3} & \ldots & 0
       \end{bmatrix}$$
     • Individual matrices concatenated into 3D matrix.
   - Landmark coordinates can be reconstructed from distance matrix using *principal coordinate analysis* (PCoA) or *multidimensional scaling* (MDS).
   - Extends to 3D distances.
(2) Triangulation:
  - Requires $2n-3$ distances among $n$ landmarks.
  - Many different triangulations possible for a configuration of landmarks.
  - E.g., 42 triangulations for 7 landmarks.
  - Can search for “best” triangulation.
    - Criterion?
- Can reconstruct point coordinates from a set of triangulated distances:
  - Lengths of three sides determine a triangle.
  - Disadvantage: actively compounds error.
(3) *Box truss*: intermediate method.

- For *coplanar points* in 2D, requires an intermediate number of distances: \( \frac{5}{2} n - 4 \)

- Within each *truss cell*, the 6 distances must be mutually compatible for the 4 landmarks to be coplanar.
  - If not, can statistically ‘relax’ them.
  - Measure *strain*: function of differences between original and relaxed distances.
  - Used to assess measurement error in terms of *mutual inconsistency* of fit.
‘Relaxing’ a truss cell:

- In practice, all landmark coordinates can be reconstructed simultaneously using principal coordinate analysis (PCoA) or multidimensional scaling (MDS).
– Extensible to 3D truss cells:
  • 22 distances among 8 points.

  ![Diagram of a 3D truss cell]

  • 4D cell ‘relaxed’ into 3D by PCoA or MDS.
  • *Strain* measures lack of mutual agreement of distances.
• Truss measurement protocol enforces:
  – Localized rather than extensive distances.
  – Evenness of orientation and coverage.
• Allows discovery of geometric relationships among characters.
• The ‘relaxation’ procedure can compensate for slight imprecision or measurement error:
  – Measurements taken to nearest 0.05 mm (higher precision).
  – Rounded to nearest 1 mm (lower precision).
• Strain estimates can be used to assess measurement error:
• Trusses and allometry:
  – Allometric coefficients are log-log regressions of characters on some measure of body size:

\[
\log(\text{Body size}) \quad \text{vs} \quad \log(\text{Character value})
\]

\[
slope = 0.47
\]
• Trusses and allometric coefficients:

Sculpins

Lake forms

Stream forms

Difference

Piranhas
- Trusses and allometry: ‘growing’ the truss

\[ a_1 = 0.35 \]

\[ \log(d_1) \]

\[ a_2 = 0.36 \]

\[ \log(d_2) \]

Etc. for all truss distances
• Trusses and morphospace:
  – Trusses can be reconstructed from predicted interlandmark distances.
  – *Strain* measures the incompatibility of the predicted truss distances.
Mapping trussicules onto trees
Trussicule evolution

Trussicule diversification
Mapping other forms onto trees using all pairwise distances among landmarks:

Hawaiian drosophilid flies
• It’s common to use ‘adulterated’ truss patterns:
  – Combinations of truss cells, triangles, isolated distances, etc.

Juste et al. (2001)